any case, if the locked-in lateral stresses were not negligible they would be expected to make the specimen less compressible rather than more compressible as was observed.

The writers agree with Ghaly that degree of saturation has a strong effect on the degree of collapse strain developed. However, for degree of saturation above 85% the collapse is essentially complete and is insensitive to degree-of-saturation changes. Both the block specimens and the disturbed specimens were above 85% after wetting.

Free Vibration of Embedded Foundations: Theory versus Experiment[a]

Discussion by Mark Svinkin[3]

The authors present experimental results to verify the homogeneous–half-space solutions for dynamic stiffnesses and damping factors derived by Gazetas (1991) in the convenient form of algebraic formulas and dimensionless charts. The conditions of the experiments for verification study chosen by the authors were as close to the assumptions of the theory as possible. Free-vibration experiments were conducted in a bin on 54 small-scale models with footing area of 108 sq in. (0.07 m^2) and 27.2–27.9 sq in. (0.018 m^2). Average damped natural frequencies for vertical model oscillations were within 72–93 Hz. The authors had certain difficulties in the study of vertical motions. The use of spectrum analysis would simplify the determination of these frequencies with greater accuracy. Theoretical and experimental results coincided very well, but the field for practical applications of the obtained solutions is not addressed, in particular for what foundation sizes these solutions can be used.

The relationship established by Tsytovich (1973) between a foundation settlement and its footing area is of great interest. The generalized curve of numerous tests on the study of soil settlements for the same pressure on soil and various loading areas is displayed in Fig. 6. There are three fields on this curve. Field I is a small loading area (approximately less than 0.25 m^2) where for average pressures, the soil is in the shear phase and settlement decreases with increasing the area. Field II is for loading areas from 0.23–0.50 m^2 to 25–50 m^2 settlements are strictly proportional to $F^{0.5}$ and correspond to the compacting phase. And field III is for areas that are bigger than 25–50 m^2 where actual settlements are less than the calculated ones. The mentioned limits can vary for loose or very compacted soils. The curve in Fig. 6 suggested that the small-scale models tested relate to field I and behaved like piles rather than foundations.

The relationship between foundation settlement and its footing area derived for static loads is valid, in general, for dynamic loads. Still, Barcan (1962) found that there are some discrepancies between the homogeneous–

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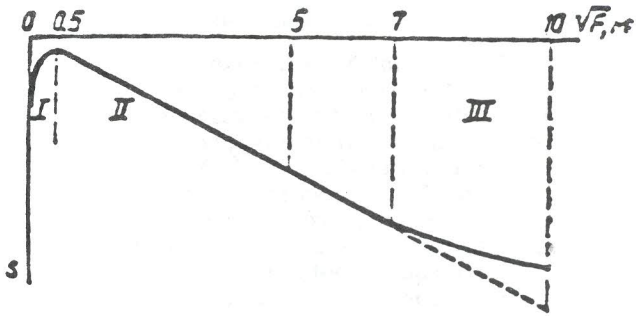

FIG. 6. Relationship between Settlement of Natural Soils (s) and Sizes of Load Area (F) [after Tsytovich (1973)]

half-space solutions and experimental results. Novak (1985, 1987) reported similar phenomena.

Dynamic properties of foundation-soil systems depend on footing area, and are completely different from the tested small models. It was confirmed experimentally for the permissible vibration level that machine foundations with areas exceeding 10 m^2, set at natural base, can be considered as linear systems. Nonlinear oscillations are displayed only for foundations with areas smaller than 10 m^2 (Svinkin 1975, 1991). Damped natural frequencies of vertical foundation vibrations decrease with increasing footing area. For actual vertical dynamic loads on foundations (Svinkin 1980) and large areas, greater than 100 m^2, these frequencies become equal to a few hertz (Svinkin 1976).

What is the range of real foundations for which suggested solutions can be used? Nobody knows. It is suggested that the authors continue their interesting study to determine the practical application and limitations of the homogeneous–half-space theory for real machine foundations.

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Closure by George Gazetas[4] and Kenneth H. Stokoe II[5]

The writers would like to thank Mark Svinkin for his interest in the paper and for his thought-provoking comments regarding the possible effect of the size of a footing on its vertical dynamic response.

His arguments stem from two different types of empirical data, based on research conducted in the Soviet Union by several researchers over many years. The first set of data refers to experiments with statically loaded surface foundations; these data are summarized in Fig. 6 [from Tsytovich (1973) and (1976)] in the form of a (presumably average) relationship between foundation settlement and square root of foundation-soil contact area. The second set of data comes from full-scale vertical vibration tests conducted by Barkan and coworkers (and reported in, among other places, Barkan's 1962 English-translated book), who observed that the natural frequency of a foundation block decreases with increasing area of the foundation-soil contact surface.

Svinkin implicitly suggests that the foregoing trends, observed in the static and dynamic Soviet field tests, are not consistent with the homogeneous–half-space theory, such as the one tested against small-scale experimental data in the discussed paper. And he quite legitimately raises the question of the range of applicability of the viscoelastic homogeneous–half-space solutions presented in the companion paper (Gazetas 1991) and calibrated in the discussed paper. Our answer is as follows.

General Background

There are primarily two reasons limiting the applicability of the viscoelastic homogeneous (and isotropic) soil model: (1) The actual soil profile may be far from homogeneous; and (2) the magnitude of the applied (static or dynamic) loads may be large and thereby produce strong local or global nonlinearities in the soil. The engineer must have a clear understanding of these two aspects of the problem (i.e. nature of soil profile, relative intensity of loading) before deciding to use the presented or a different theory, and before selecting an appropriate value of soil modulus.

The homogeneous–half-space solutions are not a panacea. Indeed, solutions have also been published for several other idealized soil profiles [e.g., Luco (1974), Kausel and Roësset (1975), Gazetas (1983) and (1991b), among others] within the realm of linear theory. Moreover, numerical algorithms and computer codes have been developed for handling more general soil profiles (e.g., the codes FLUSH, CLASSI, SASSI, etc.). Particularly important, for example, is the presence of a stiff ("rock") base at relatively shallow depths (or for a given depth, with foundations of large dimensions). Such a base may exert a profound effect on both stiffness and damping of the foundation-soil system.

Fewer studies have been published to account for nonlinear plastic soil behavior under an oscillating foundation. For important projects the current state of practice, in the U.S. at least, is to resort to iterative "equivalent-linear" analyses using soil moduli G and damping ratios β (for each soil element) that are consistent with the level of strain γ resulting from the previous analysis. In most cases, however, an additional simplification is considered practically sufficient (if not computationally necessary): a single

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representative set of values is selected for the modulus and damping of each soil layer, rather than element, using semiempirical and analytical rules, such as those proposed by Whitman (1976), Jakub and Roësset (1977), and Novak (1985), among others.

Empirical Data of Tsytovich and Barkan

Unfortunately, the information contained in the aforementioned interesting results of the large-scale Soviet experiments is not sufficient to clearly delineate the effects of soil inhomogeneity (layering) from the effects of nonlinear inelastic soil deformation. Moreover, the discusser may perhaps not be fully aware of some fundamental differences in the behavior of foundations under static and under vibratory loading. Hence, his unfortunate (as is shown later in this closure) statement that the "relationship between foundation settlement and its footing area derived for static loads is valid, in general, for dynamic loads."

We begin with the static settlement curve of Tsytovich (1973, 1976), shown in Fig. 6. Without knowledge of the soil profile (e.g., in the form of the variation of soil Young's modulus with depth) and without information on the magnitude of the applied load, one can hardly draw any quantitative conclusions from Fig. 6. Nevertheless, a qualitative (and somewhat speculative) explanation of the exhibited trends can be advanced. It is presumed that the applied load(s) were of a magnitude representative of actual foundation cases, within a factor of safety of about 2 or 3 from the ultimate load that would produce bearing-capacity failure. Strong nonlinearities in certain elements of the supporting soil under such loads are unavoidable. Under the footing edges, in particular, where a rigid foundation on the surface of an elastic continuum would tend to produce extremely large normal contact stresses, the lack of confinement (due to the presence of the nearby free surface) initiates very severe soil yielding and thereby the edge stresses are dulled and redistributed. Thus, the "effective" width of the foundation is reduced.

The smaller the size of the footing, the greater the significance of this reduction in effective width. Footings with sizes falling in region I of Fig. 6, in particular, act like small rigid "stamps" and produce contact pressures that are parabolically distributed, with a maximum value at the center (Kerr 1989). In this region, with decreasing footing size not only is the effective width reduced disproportionately but, also, the parabolic contact-stress distribution becomes increasingly sharper; hence the observed increasing settlement, if everything else had remained the same.

With larger footing sizes (regions II and III), the effect of such a redistribution of edge stresses becomes insignificant and, as Kerr (1989) emphasized, even surface foundations on sand would develop a saddle-shaped contact stress distribution, similar to that predicted for an elastic continuum. The linear relationship of settlement versus square root of contact area for region II in Fig. 6 should thus not surprise. On the other hand, two phenomena may be responsible for the flattening of the settlement curve for the large foundations of region III: (1) The stiffer soil layers that are quite likely to have existed at greater depths in the tested site(s) fall now within the pressure bulb of the loading footing, and lead to decreasing settlements; and (2) soil nonlinearities may be less pronounced under a large foundation because of more favorable confinement conditions [e.g. Novak (1985)].

Notwithstanding the previously quoted statement of the discusser, there

are at least two fundamental differences between static and dynamic (vibratory) foundation behavior.

1. The amplitude of strains induced in the soil by dynamic vertical loads will typically be about two orders of magnitude smaller than under statically loaded foundations [see, for example, Woods (1978)]. For machine foundations, in particular, the difference may be even greater, and soil nonlinearities will be of marginal importance in most cases. Moreover, "edge" effects will be of little, if any, consequence. (Among the possible exceptions is rocking-swaying of footings in soft soil under strong seismic excitation, which, however, is beyond the scope of the discusser's comments.)
2. The waves that emanate from the contact surface of an oscillating foundation propagate to large depths and horizontal distances. They are thus affected by the (unavoidable) presence of stiffer soil layers at great depths, and this is reflected in the footing response. By contrast, statically loaded foundations are affected by soil at more limited depths.

As a result of these two fundamental differences the settlement–versus–area-of-footing relationships under static and under vibratory loading are, generally, quite different.

Several additional questions arise relating to Barkan's empirical findings as presented in Svinkin's discussion and as reported by Barkan (1962). The most important is that the actual soil profile and the variation of shear modulus with depth not reported, and are presumably only poorly (if at all) known. Therefore, attempts to rationalize and draw general, purely empirical conclusions from such experiments are bound to fail.

Nonlinearities in Erden's Tests?

The final question that must be answered is to what extent soil nonlinearities had "contaminated" the experimental results used in our study. We examine separately the static and the dynamic phases of the experiments, with reference to Erden (1974).

First, the three statically applied weights (183 lb, 240 lb, and 309 lb) induced average contact stresses on the soil-footing interface of about 1.67 psi, 2.18 psi, and 2.80 psi, respectively (or about 11.5 kPa, 15.0 kPa, and 19.4 kPa). These are too small stresses for any substantial nonlinearities to develop in the dense sand ($D_r \approx 93\%$) of the test bin. Indeed, Erden [(1974) page 98] reports

> Finally, all the attached weights were removed and the footing was retested to determine whether the additional weights imparted any permanent effects on the response of the soil-footing system. In no case was any measurable difference observed before and after the attachment of the weights.

Note furthermore that because of the embedment of the tested footings, the edge contact stresses would not tend to reduce to zero, as is the case with surface footings, such as those utilized by Tsytovich (1976). The increased confinement of the sand directly under the edges increases its strength; experimental measurements [summarized by Kerr (1989)] show that edge stresses may indeed attain very substantial values, in rough proportion to

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the depth of embedment. Therefore, little redistribution of stresses would have taken place under the vertical gravity loads of the tested footings.

In any case, what is of major importance is whether nonlinearities developed under the dynamic ("additional") loading—not under the static ("existing") load. In view of our earlier observation that dynamic loading is associated with strains smaller by about two orders of magnitude than vertical gravity loading, one would not expect nonlinear soil effects on the measured natural frequencies and damping ratios. Indeed, Erden [(1974) page 100] reports

> ... for each mode of vibration, different magnitudes of load (varying between 1 lb and 15 lbs) were imposed to investigate whether this factor affected the response of the soil-footing system. In no case was any measurable effect observed.

In conclusion, the writers believe they were justified in modeling the soil in Erden's experiments as a linear (viscoelastic) homogeneous half-space. But this by no means should be construed as an endorsement of the indiscriminate use of this model in all practical applications.

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